THE WRITING OF BRAILLE MATHEMATICAL NOTATION

A Guide to the Taylor System of Mathematics

By

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Taylor System of Mathematics

MADELEINE SEYMOUR LOOMIS

Education of the Exceptional

TEACHERS COLLEGE
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PREFACE

The Taylor System is used by the English-speaking countries for the writing of mathematics in Braille. The system was devised by the Reverend William Taylor of York, England, in 1836.

This book presents the Taylor System without changes. It does, however, include the supplementary rulings of Mr. John R. Emblen which appear in his Braille book *A Guide to the Writing of Arithmetic, Algebra and Mathematical and Chemical Formulae in Braille*, published by the National Institute for the Blind, London. Mr. Emblen gives more detailed and very necessary rulings in regard to the use of brackets.

In 1932, when Standard English Braille was adopted as the system for general literature for the English-speaking countries, the dollar sign was changed. Braille editions of mathematical textbooks embossed prior to this date will show the dollar sign as the letter *d* preceding the numeral sign. The present method of writing this sign is *lower d* followed by the numeral sign. In 1932, the method of numbering the dots was altered; a change that has been adopted by both Great Britain and America. The former method of numbering was 1-3-5 for the right-hand dots and 2-4-6 for the left-hand dots. The present numbering is 1-2-3 for the right and 4-5-6 for the left-hand dots. The present method of numbering is used in this book, but Taylor's book of signs employs the former. The customary capital sign, which is always used in American texts, appears in this book.

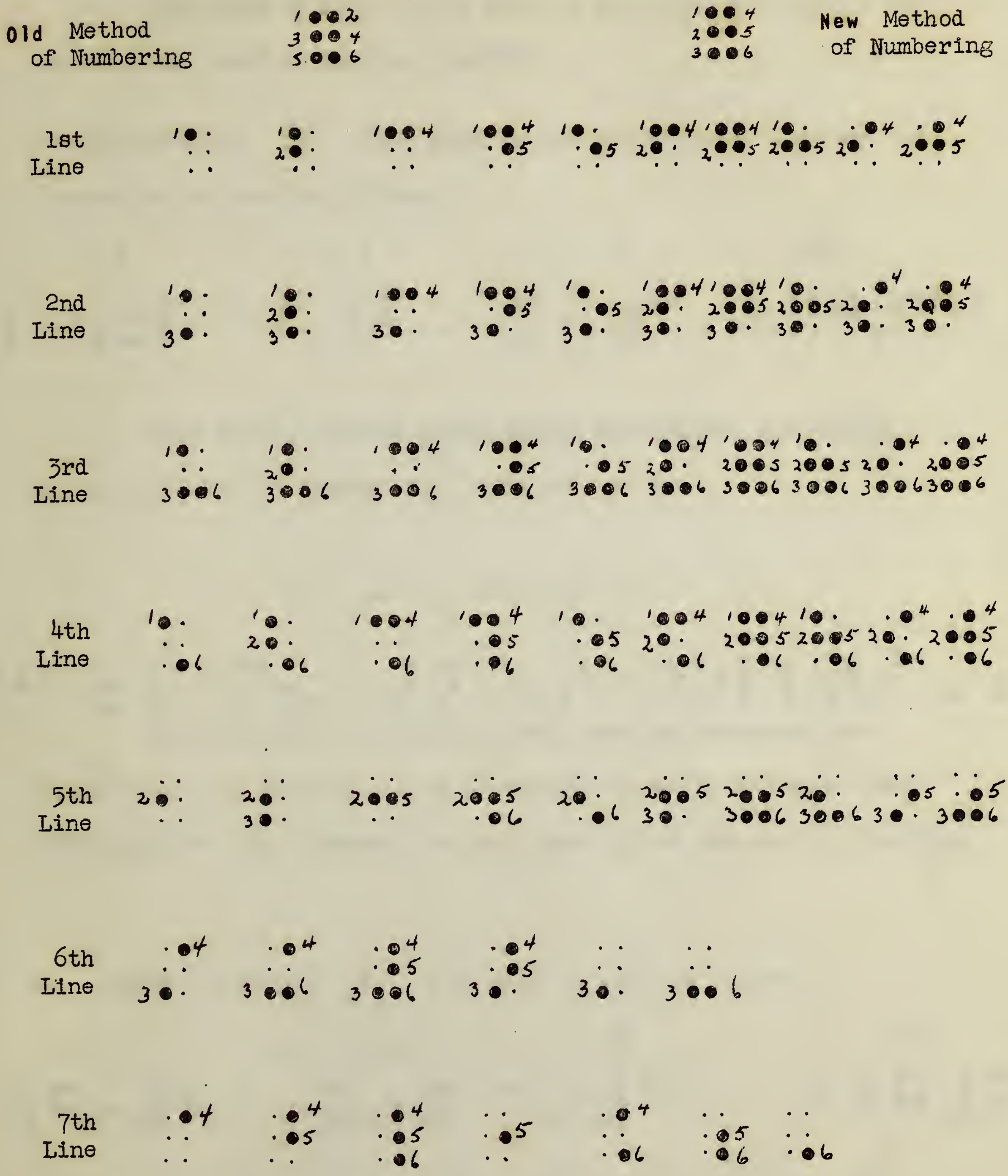
PREFACE

The Taylor System is used by the English-speaking countries for the writing of mathematics in braille. The system was devised by the Reverend William Taylor of Bath, England, in 1836.

This book presents the Taylor System without changes. It does, however, include the supplementary rulings of [illegible] Clarke which appear in his braille book, *A Guide to the Writing of [illegible]* published by the National Institute for the Blind, London. Mr. Clarke gives more detailed and very necessary rulings in regard to the use of brackets.

In [illegible] when Standard English braille was adopted as the symbol for [illegible] for the English-speaking countries [illegible] the [illegible] and new [illegible]. [illegible] of [illegible] this book will show the [illegible] sign, or the [illegible]. The [illegible] of [illegible] is [illegible] in the [illegible]. In [illegible], the method of [illegible] that had been adopted by both Great Britain and America. The former method of [illegible] was 1-4-6 for the [illegible] and 2-4-6 for the [illegible]. The present [illegible] is 1-4-5 for the [illegible] and [illegible]. The present method of [illegible] is used in this book, but [illegible] the former. The [illegible], which is widely used in American [illegible], appears in this book.

THE SIXTY-THREE BRAILE SIGNS ARRANGED IN SEVEN LINES



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BRAILLE MATHEMATICAL NOTATION

The signs of line 1 (see page 2) are, when preceded by the numeral sign, used to express numerals.

1. **PLUS**--dots 2-6 This sign is not preceded nor followed by a space but is used as follows:

$5 + 2$ $a + b$ $x + 3$ $+30^{\circ}$

Only single letters representing algebraical quantities are preceded by the letter sign, dots 5-6, as in:

If A men earn B dollars ...

The plus sign in print is often used to represent the positive but, in Braille, it is preferable to write the word like "The positive* pole" rather than the word "pole" preceded by dots 2-6.

2. **MINUS** --dots 3-5 This sign is used as follows:

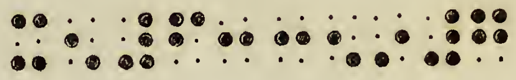
$7 - 2$ $x - y$ $a - 31$ -60°

The minus sign in print is often used to represent the negative but, in Braille, it is preferable to write the word like "The negative* main" rather than the word "main" preceded by dots 3-5.

3. **PLUS OR MINUS** --dots 2-6 followed by dots 3-5 This sign is used as follows:

± 7

± 18

$x + 6 = \pm 7$ 

4. **MINUS OR PLUS** --dots 3-5 followed by dots 2-6 This sign is used as follows:

c minus or plus d

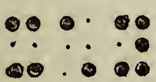
$c \mp d$

5. **DIFFERENCE BETWEEN** --dots 2-5 followed by dot 3, This sign is used as follows:

The difference between x and y

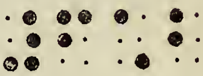
$a \sim b$

6. **MULTIPLIED BY** --dots 1-6 This sign is used as follows:

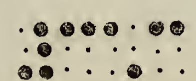
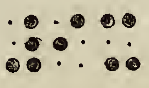
x x y 

12 x 15

a x b

6 x b 

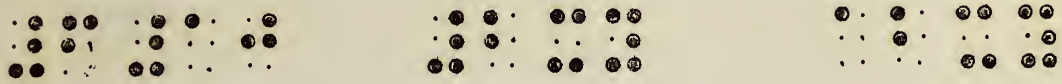
When an algebraical quantity represented by a Braille sign of line 1 (see page 2) has a numerical coefficient, the sign dots 1-6, must be inserted after the number. In other cases it may be omitted.

Examples: 4a 6b 3c  9x  12y

The sign is necessary to prevent the first three examples from being read as 41, 62, and 33.

When one quantity (algebraical or numerical) is written close after another, it means that the first is multiplied by the second as:

6 x 10   2 x x x y   a x b x x x y



In inkprint, a dot--placed higher than a decimal sign--is sometimes used to express multiplication as:

7·2a   (meaning 7 x 2a)

Though inadvisable, it is often necessary to use this sign between words in which case the sign, dots 1-6, should be repeated as:

length x breadth

7. **DIVIDED BY**--dots 3-4, or dots 3-4 followed by dots 3-4, or

This sign is used as follows:

$x \div y$ or $\frac{x}{y}$ $y = \frac{1}{x}$ $\frac{ab}{cd} \div \frac{bc}{ad}$

$\frac{4}{5}$ $4 \div 5$

(Note the difference in the writing of these two examples.)

This sign for "divided by" will be recognized as the fraction-line sign and for this reason it is often necessary, in Braille, to distinguish between the "fraction-line" and the "divided by" signs. Four-fifths and four divided by five may have the same value but, if used in conjunction with an algebraical quantity are widely different.

Example:

$\frac{4}{5}y$

$\frac{4}{5y}$

$\frac{4}{3b}$

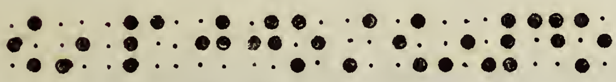
(The difference is denoted by the repetition of the numeral sign.)

Note the use of the sign in the following example to prevent three divided by b from being read as the fraction $\frac{3}{2}$:

$3 \div b$

Additional examples showing the use of the division sign:

$(+100) \div (-4)$



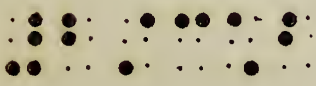
$36n \div 12$

$\frac{2}{3}x$

$\frac{2}{3x}$

(Note repetition of the numeral sign.)

$\frac{2}{3}b$



$\frac{2}{3b}$

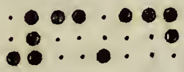
(Note repetition of the numeral sign.)

8. FRACTIONS

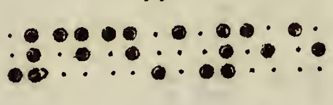
There are three ways of writing a fraction whose numerator and denominator are both numbers. The first two are as follows:

$\frac{3}{4}$

or



$\frac{44}{55}$



or

The third method which is sometimes used is as follows:

$\frac{4}{5}$

$\frac{7}{8}$

$\frac{44}{55}$

$\frac{53}{85}$

In using this third method extreme caution is required in order to prevent the denominator or last figure in the denominator from being taken for an algebraical sign.

$\frac{4}{5}y$ might be read as $4 + y$ if written

In order to save space, the following are sometimes written as follows:

$\frac{1}{2}$

$\frac{1}{3}$

$\frac{1}{4}$

$\frac{1}{5}$ etc.

The sign must be distinguished from

Examples:

$\frac{2}{4} \div \frac{6}{9}$

$\frac{7}{8} \div \frac{1}{3}$

$4 \div b$

$12 \div c$

<u>When the numerator or the denominator of a fraction consists of two or more terms, it must, in Braille, be enclosed in brackets</u>; should bracketed portions already exist, a second pair must be introduced. Simple or complex fractions may, in letterpress, be written without the actual use of brackets, but they cannot be so written in Braille. (See "Brackets," page 14).

Example: $\frac{a + b}{a - b}$

(If the brackets in the above example had been omitted, it would read which is a trinomial expression, the middle term of which is the fraction $\frac{b}{a}$.)

$$\frac{an + bn - cn}{ap + bp - cp}$$

9. **IS EQUAL TO**--dots 2-5 followed by dots 2-5, This sign is used as follows:

a = b

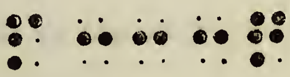
4 + 4 = 8

9 - 2 = 7

Unless the saving of space is of great importance, a space should be left on either side of this sign when used between words as:

area = length x breadth

10. **IS NEARLY EQUAL TO** --dots 2-5 followed by dots 2-5 followed by dots 2-5, This sign is used as follows:

p is nearly equal to q 

11. **IS NOT EQUAL TO**--dots 2-5 followed by dots 2-4-5-6,

This sign is used as follows:

c is not equal to d

$c \neq d$

12. **IS IDENTICALLY EQUAL TO**--dots 2-3-5-6 followed by

dots 2-3-5-6, ⠶⠶ This sign is used as follows:

x multiplied by y divided by x is identically equal to y

$x \times y \div x \equiv y$

⠭⠡⠽⠌⠭⠶⠶⠽

13. **IS GREATER THAN**--dots 2-5 followed by dots 1-4, ⠒⠉

This sign is used as follows:

p is greater than q $\qquad\qquad \frac{a + b}{2} > \sqrt{ab}$

$p > q$

⠏⠒⠉⠟ $\qquad$ ⠪⠁⠢⠃⠕⠌⠼⠃⠒⠉⠜⠪⠁⠃⠕

In order to avoid confusion, it is often useful to separate the signs of algebraical operation from the quantities which are affected by them.

Example: $a > b$

⠁ ⠒⠉ ⠃

Without the space this might be read as the ratio of a to cb ⠁⠒⠉⠃ (See "Ratio," page 16.)

14. **IS NOT GREATER THAN**--dots 2-5 followed by dots 1-4-5-6, ⠒⠹

This sign is used as follows:

x is not greater than y ⠭⠒⠹⠽

15. **IS GREATER THAN OR EQUAL TO**--dots 2-5 followed by dots 1-2-4-5, ⠒⠛

This sign is used as follows:

x is greater than or equal to y ⠭⠒⠛⠽

$x \geqq y$

16. **IS LESS THAN**--dots 2-5 followed by dots 3-6, ⠒⠤ This sign is used as follows:

m is less than n

$m < n$ ⠍⠒⠤⠝

17. **IS NOT LESS THAN**--dots 2-5 followed by dots 3-4-5-6, ⠒⠼ This sign is used as follows:

p is not less than q ⠏⠒⠼⠟

18. **IS EQUAL TO OR LESS THAN**--dots 2-5 followed by dots 2-3-5-6, ⠒⠶

This sign is used as follows:

p is equal to, or less than q

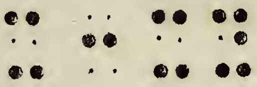
$p \leqq q$ ⠏⠒⠶⠟

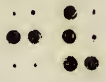
19. **IS GREATER THAN, EQUAL TO, OR LESS THAN**--dots 2-5 followed by dots 1-2-3-4-5-6, This sign is used as follows:

x is greater than, equal to, or less than y

20. **IS GREATER OR LESS THAN**--dots 2-5 followed by dots 1-3-4-6,

This sign is used as follows:

x is greater or less than y 

21. **VARIES AS**--dots 2-5 followed by dots 1-2-3-6, 

This sign is used as follows:

a varies as b

$a \overset{*}{\propto} b$

22. **SINGLE LETTERS**, when representing algebraical quantities, are preceded by the letter sign, dots 5-6, Example:

If A men work B days, etc.

etc.

23. **GREEK LETTERS** are preceded by dot 3, They are written as follows:

$2\pi r$

* This sign is now obsolete but will be found in some books.

INTERNATIONAL GREEK ALPHABET*

Α	α	alpha	⠄⠁
Β	β	beta	⠄⠃
Γ	γ	gamma	⠄⠛
Δ	δ	delta	⠄⠙
Ε	ε	epsilon	⠄⠑
Ζ	ζ	zeta	⠄⠵
Η	η	eta	⠄⠱
Θ	θ	theta	⠄⠹
Ι	ι	iota	⠄⠊
Κ	κ	kappa	⠄⠅
Λ	λ	lambda	⠄⠇
Μ	μ	mu	⠄⠍
Ν	ν	nu	⠄⠝
Ξ	ξ	xi	⠄⠭
Ο	ο	omicron	⠄⠕
Π	π	pi	⠄⠏
Ρ	ρ	rho	⠄⠗
Σ	σ ς	sigma	⠄⠎
Τ	τ	tau	⠄⠞
Υ	υ	upsilon	⠄⠥
Φ	φ ϕ	phi	⠄⠋
Χ	χ	chi	⠄⠯
Ψ	ψ	psi	⠄⠽
Ω	ω	omega	⠄⠺

* The dot 3 is used with the character only when it appears in mathematical notation.

24. **DASHED LETTERS** are followed by dots 3-6, Examples:

$\frac{x'}{y'}$ $\frac{B'C'}{A'C'}$

1'74'24

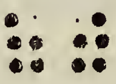
(The numeral sign is not repeated)

25. **INFINITY**--dots 1-2-3-4-5-6,

∞

26. **PAIRS OF BRACKETS:**

dots 2-4-6 and dots 1-3-5,

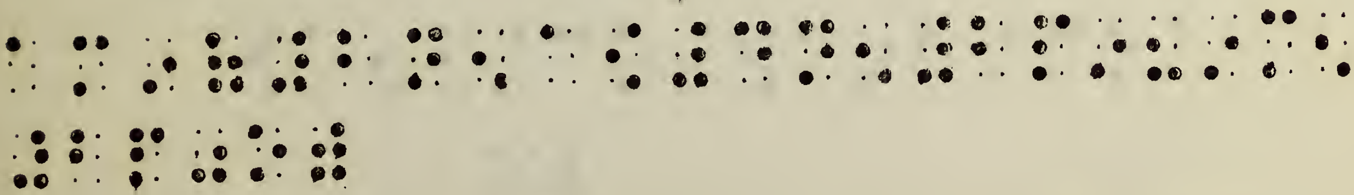
dots 1-2-3-5-6 and dots 2-3-4-5-6, 

dots 2-3-6 and dots 3-5-6,

dots 2-3-5-6 and dots 2-3-5-6,

(The fourth form of brackets is seldom used mathematically.)

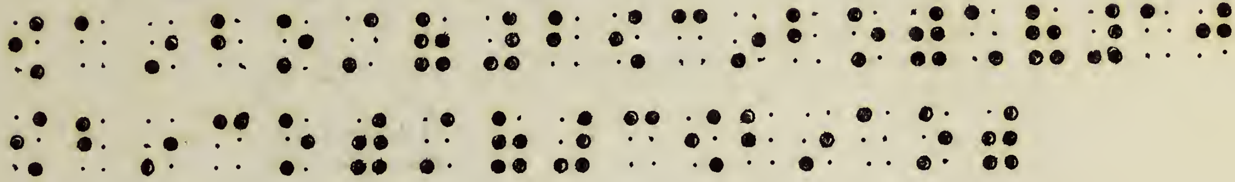
Ex: $am - \left[2n + a \left\{ 4n + 2p - (-m + 2p) \right\} \right]$



Simple or complex fractions may, in print, be written without the actual use of brackets, but they cannot be so written in Braille. Brackets are of utmost importance in accurate reproduction and the avoidance of ambiguity. When the numerator or denominator of a fraction consists of two or more terms, it must, in Braille, be enclosed in brackets; should bracketed portions already exist, a second pair must be introduced. (See Fractions, page 7.)

Examples: $\frac{5(x + y)}{7(x - y)}$

$\frac{a - b}{2(c - b)} \cdot \frac{10(b - c)}{3(b - a)}$

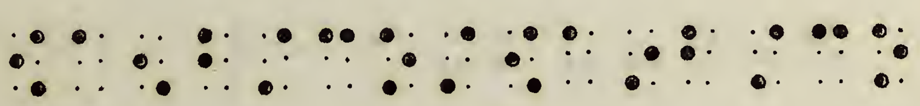


Notice the second pair of brackets, which does not appear in the printed version, that are necessary when it is written in Braille.

FRACTIONS

In the writing of fractions, especially complex fractions, much depends upon the correct use of the brackets. In the third and fourth examples which follow, note well the addition of the sets of brackets which do not appear in the print.

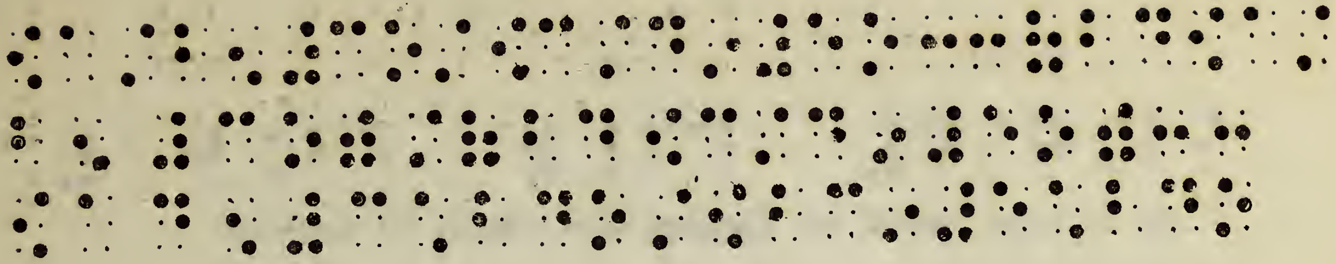
1. $$\frac{a + \frac{b}{c}}{a - \frac{b}{c}}$$

 *

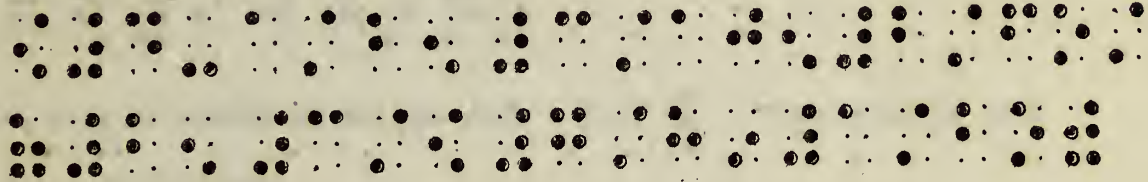
2. $$\frac{m - \frac{4}{m}}{1 + \frac{2}{m}}$$

* There is no danger of this being read as $\frac{a - b}{c}$ because it would then be written $\left(\frac{a - b}{c}\right)$

3. $$\frac{\frac{a}{b} + 3}{\frac{c}{d} - 5} = \frac{bd(\frac{a}{b} + 3)}{bd(\frac{c}{d} - 5)} = \frac{ad + 3\,bd}{bc - 5\,bd}$$

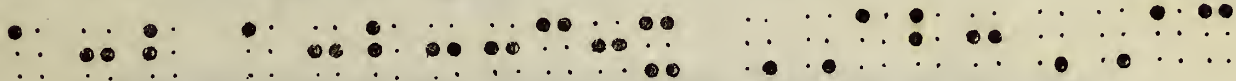


4. $$\frac{4\frac{1}{2} + \frac{3}{10} + \frac{2}{6}}{2 + \frac{3}{\frac{7}{8} - \frac{1}{2}}}$$



27. **RATIO**--dots 2-5 This sign is used as follows:

a : b a : b = c : x AB : AC

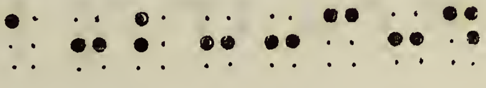


28. **PROPORTION**--dots 2-5 followed by dots 2-5,

This sign is used as follows:

a is to b as c is to d

a : b :: c : d



29. **LONG DIVISION: AFTER DIVISOR**--dots 1-3-5,

BEFORE QUOTIENT--dots 2-4-6,

This sign is used as follows:

$$12\overline{)1488}\quad 124$$

30. **SIGN OF THE DECIMAL POINT**--dots 4-6, ⠨ This sign is used as follows:

10.25 .64 497.29

Note that the numeral sign <u>precedes</u> the decimal point in .64.

The decimal point should not be confused with the multiplication sign which is sometimes expressed by a single dot.

.06(900 - n)

31. **SIGN OF RECURRING PERIOD**--dots 4-5-6, ⠸ This sign is used as follows:

$= 3 . \dot{3}$

When the whole decimal fraction recurs, both the decimal point and recurring period sign must be inserted.

$8.1\dot{6} \times 3\cdot\dot{1}4285\dot{7}$

32. **ROOT SIGN**--dots 1-2-4-5-6, ⠻ This sign is used as follows:

$\sqrt[N]{n}$ $\sqrt[5]{32}$

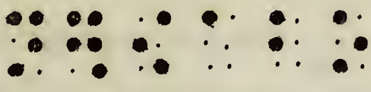
Square and cube roots are not expressed with the root sign as there are special signs to represent them. (See pages 17, 19, 21.)

Whole numbers do not require brackets but fractions should be bracketed.

Examples: $\sqrt[6]{45}$ $\sqrt[6]{\frac{4}{5}}$

When the whole of an expression (algebraical or numerical) consisting of two or more factors or terms is to be affected by the root sign, it must be enclosed in brackets; if any part of such expression is already bracketed, a second pair should be introduced.

Examples: $\sqrt[n]{ab}$ $\sqrt[n]{2a}$



$\sqrt[n]{a - b}$ $\sqrt[n]{2(a - b)}$

(For the writing of $\sqrt[n]{a^{1}}$, see "Square Root", page 19.)

Note the difference in the writing of the two examples which follow:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

33. **SQUARE**--dots 1-2-6, This sign is used as follows:

3^2 a^2 $7^2 = 49$

xy^2 $(xy)^2$ $(12\frac{1}{2})^2$

$(a + b)^2 = a^2 + 2ab + b^2$

34. **SQUARE ROOT**--dots 3-4-5 This sign represents only the square root so that the numeral 2 is never required to distinguish it from some other root. This sign should always be used in preference to the numeral 2 followed by the root sign. (In using the Calculating Board, there is no special way to express the square and cube roots. The numerals must be added to the radical sign.)

Examples: $\sqrt{14}$ $\sqrt{x}$ $\sqrt{a^2}$

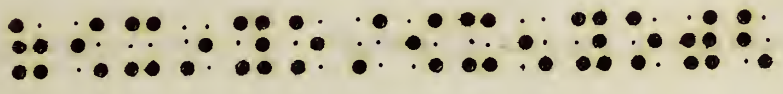
When the whole of an expression (algebraical or numerical) consisting of two or more factors or terms is to be affected by a root sign, it must be enclosed in brackets; if any part of such expression is already bracketed a second pair should be introduced.

Ex.: $\sqrt{ab}$ $\sqrt{3a}$

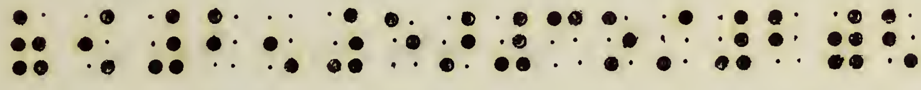
(a)

(b) $\sqrt{9 + 4}$ $\left(5 \sqrt{2a}\right)^2$

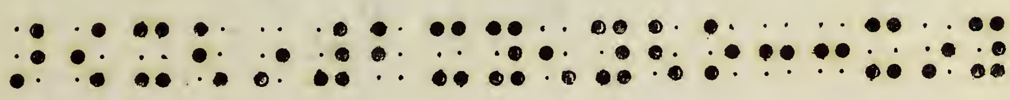
(c) Brackets are needed for fractions but not for whole numbers. $\left(\frac{x - y}{x + y}\right)^2$

 *

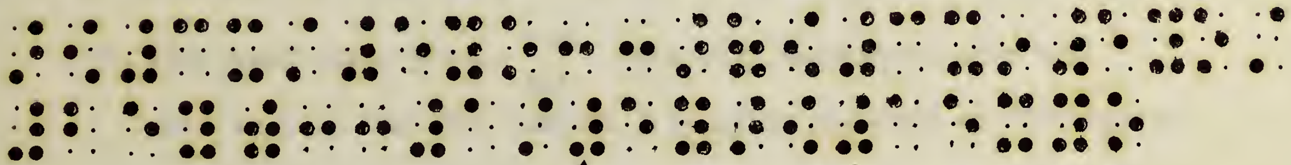
(d) $\left(\frac{2 + 5\sqrt{3}}{2}\right)^2$



(e) $\sqrt{x^2 - 2xy + y^2} = x - y$



(f) $\sqrt{\frac{3x}{5y}} = \sqrt{\frac{3x - 5y}{25\ y}} = \frac{1}{5y}\sqrt{15xy}$



(Notice the repetition of the numeral sign to prevent this from being read as $\frac{1}{5}$ y).

(g) $\sqrt{\frac{3}{4}} - 9\sqrt{\frac{1}{3}} - \frac{1}{2}\sqrt{27}$

---

*The numerator and denominator consist of two terms and must, therefore, be bracketed separately and then, the whole being affected by the index, must be enclosed in a second pair.

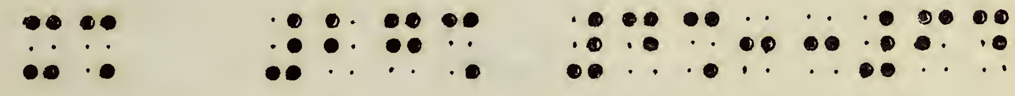
(h) $$\frac{\sqrt{x^2 + 2xy + y^2}}{x - y}$$

(i) $$\sqrt{\frac{x^2 + 2\ xy + y^2}{x - y}}$$

In examples h and i notice the use and location of brackets: that in the first, the square root applies only to the numerator and, in the latter, that it applies to both the numerator and denominator.

35. CUBE--dots 1-4-6, This sign is used as follows:

x^3 $\quad 27^3 \quad$ $4^3 = 64$



$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$3r\sqrt{\frac{r}{2}} + \frac{1}{2}\sqrt{2r^3}$$

36. CUBE ROOT--dots 3-4-6, This sign represents only the cube root so that the numeral 3 is never required to distinguish it from some other root. This sign should always be used in preference to the numeral 3 followed by the root sign. (In using the Calculating Board, there is no special way to express the square and cube roots. The numerals must be added to the radical sign.)

Examples: $\sqrt[3]{x}$ $\quad \sqrt[3]{27} = 3 \quad$ $\sqrt[3]{a^3}$

$$\sqrt{\frac{x^2+2xy+y^2}{x+y}} \qquad (c)$$

$$\sqrt{\frac{x^2+2xy+y^2}{x+y}} \qquad (d)$$

In examples (c) and (d) notice the use and function of termination sign. In the first, the radical sign applies only to the numerator and, in the latter, that it applies to both the numerator and denominator.

19. [illegible] (---), Notation is read as follows:

x^2 [illegible] $(x^2 - y)$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

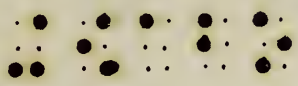
[illegible]

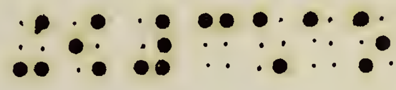
20. [illegible] (---), This sign represents [illegible] that the numeral 3 [illegible] is distinguished [illegible]. This sign should [illegible] in preference to the numeral 3 followed by the root sign. [illegible] (The [illegible] space is placed [illegible] the radical sign.)

Examples: $\sqrt[3]{x}$ [illegible] [illegible]

When the whole of an expression (algebraical or numerical) consisting of two or more factors or terms is to be affected by a root sign, it must be enclosed in brackets; if any part of such expression is already bracketed a second pair should be introduced.

Ex.: $\sqrt[3]{ab}$ $\sqrt[3]{3a}$

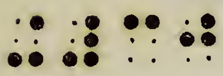


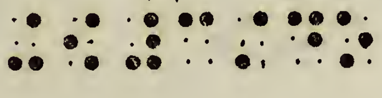


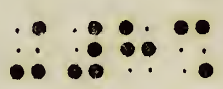
$\sqrt[3]{8 + 5}$ $\left(6\sqrt[3]{2a}\right)^2$

Brackets are needed for fractions but not for whole numbers.

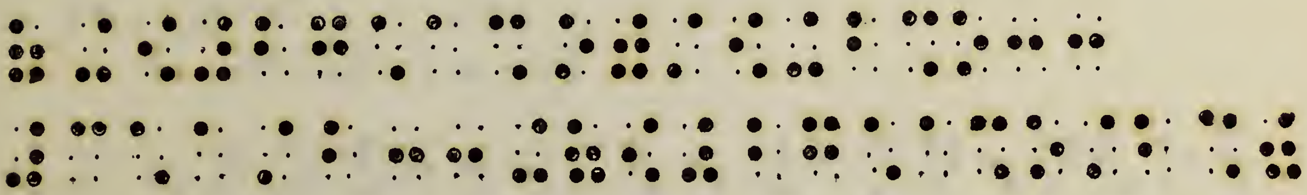
$\sqrt[3]{30}$ $\sqrt[3]{\frac{3}{4}}$ $\sqrt[3]{8^3}$







$$\frac{\sqrt[3]{27a^3}}{\sqrt[3]{b^3}} = \frac{3a}{b} = \sqrt[3]{\frac{27a^3}{b^3}}$$



$$\sqrt[3]{a^3 + 3a^2b + 3\,ab^2 + b^3} = a + b$$

Compare the example to follow with the one shown above.

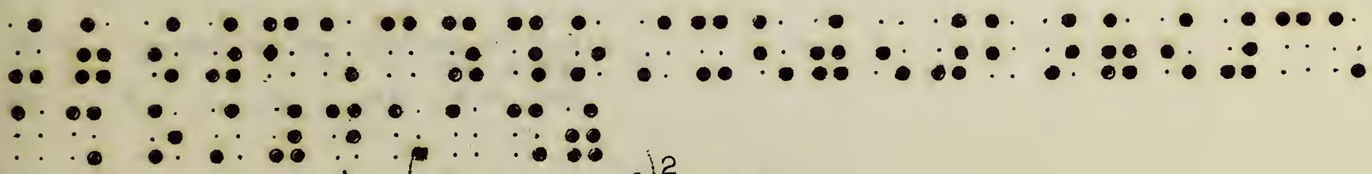
$$\sqrt[3]{a^3} \quad + \quad 3\,a^2b$$

37. **FOURTH POWER**--dots 1-4-5-6, This sign is used as follows:

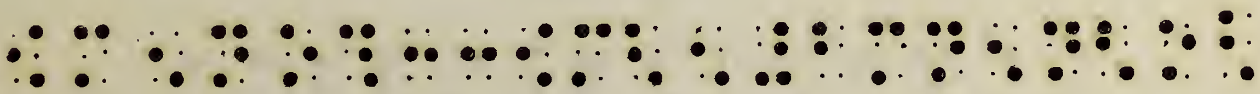
x^4

$-8\,a^3b^4$

$$\sqrt[3]{\frac{6\,cy^4}{x^2}} + 2\sqrt{\frac{3a^3}{4b^3}}$$



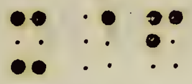
$$(m + n)^4 = \left(m^2 + 2\,mn + n^2\right)^2$$



The numeral 4 is included in the sign dots 1-4-5-6.

38. **NUMERICAL INDEX SIGN**--dot 4, ⠈ This sign changes all the Braille signs of line 1 (see page 2) which appear in the index into numbers and is used as follows:

x^6

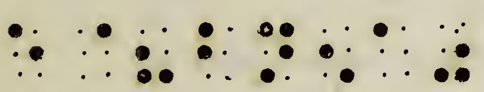


y^9

When the whole of an expression (algebraical or numerical) consisting of two or more factors or terms is to be affected by an index sign, it must be enclosed in brackets; if any part of such expression should already be bracketed, a second pair must be introduced.

Examples:

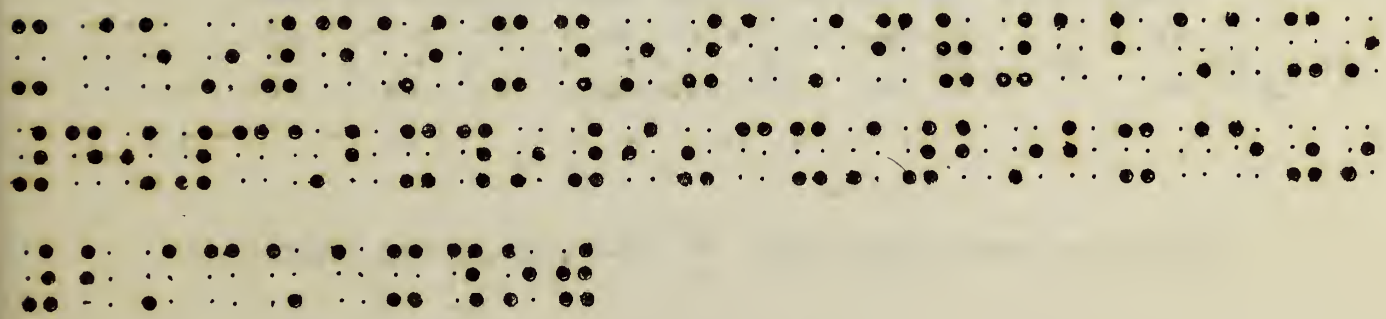
$e^{2n + 1}$



$e^{(2n + 1)^2}$

$$(a^2)^3 = a^{2 \times 3} = a^6$$

$$x^5 - 4\ bx^4 - \frac{1}{6}\left[12\ ax - 4\left\{3\ bx^4 - 9\left(\frac{cx}{2} - bx^5\right) - \frac{2}{3}\ ax^4\right\}\right]$$



$$\frac{a^3b^2}{7c^3d^4} \div \frac{a^4b^3}{14c^5d^6}$$

39. **LITERAL INDEX SIGN**--dots 4-5, This sign is used as follows:

100^n x^{m+n}

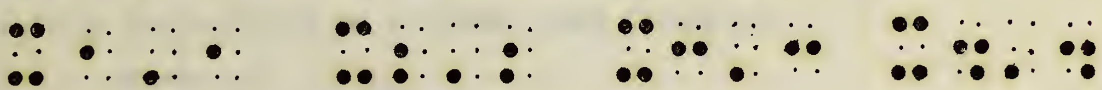
When a literal index consists of two or more terms, it must be enclosed in brackets otherwise the sign would affect only the letter which immediately follows.

40. **NUMERICAL SUFFIX SIGN**--dot 6, This sign is used as follows:

c_6 b_3 x_2 $\frac{w_1}{w_2}$

The sign changes all the Braille signs of Line 1 which appear in the suffix into numbers.

For the saving of space etc., are often written for etc., and in such cases are separated from the punctuation sign by dot 3 as:



41. **LITERAL SUFFIX**--dots 5-6, This sign is used as follows:

x_n

42. FACTORIAL SIGN --dots 2-3-5-6, ⠶ This sign is used as follows:

factorial 2 — $\underline{2|}$ or 2!

factorial 3p

(The numeral sign is not used.)

$$\frac{n(n-1)(n-2)}{3!}$$

43. NUMBER OF PERMUTATIONS OF N THINGS TAKEN R TOGETHER

$_nP_r$

NUMBER OF PERMUTATIONS OF 6 THINGS TAKEN 5 AT A TIME

44. NUMBER OF COMBINATIONS OF 10 THINGS TAKEN 5 TOGETHER

NUMBER OF COMBINATIONS OF N THINGS TAKEN R TOGETHER

$_nC_r$

45. LOGARITHM TO BASE 10

46. TABULAR LOGARITHM

47. LOGARITHM OF n TO BASE e

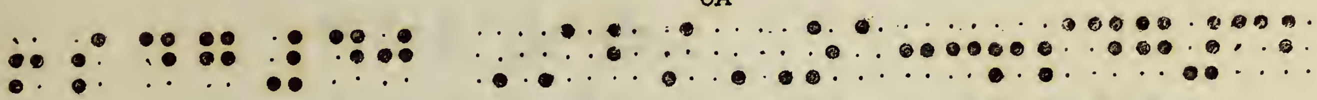
$\log_e n$

48. LOGARITHM OF 50 TO BASE 10 (or)

log 50^{10}

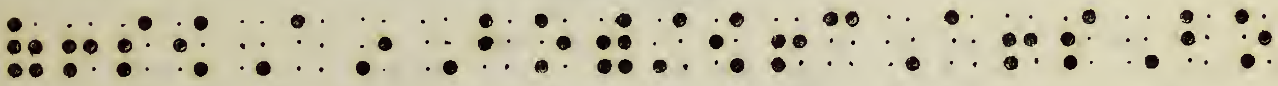
49. **SINE** (sin)--dots 2-3-5 followed by dots 2-3-4, This sign is used as follows:

$\sin 40^\circ$ $\qquad$ $\frac{AB}{OA} = \sin 32^\circ$



50. **COSINE** (cos)--dots 2-3-5 followed by dots 1-4, This sign is used as follows:

$$\frac{\sin (A - B)}{\cos A \cos B}$$



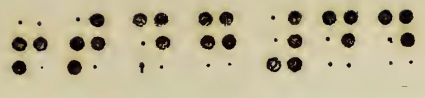
$$r \cos (\phi - \theta) = p$$

51. **TANGENT** (tan)--dots 2-3-5 followed by dots 2-3-4-5,

This sign is used as follows:

$$\frac{\sin A \pm \sin B}{\cos A + \cos B} = \tan 1/2 (A \pm B)$$

tan 44°



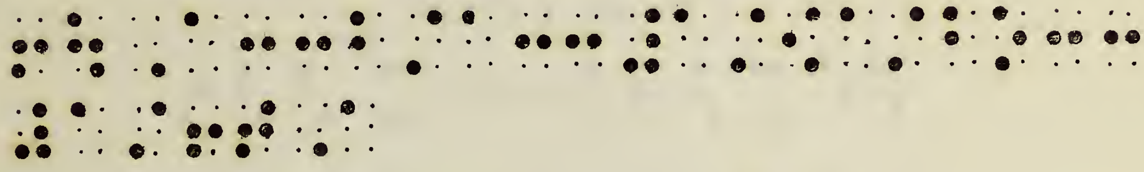
tan 31° $30'$

*

52. **COTANGENT** (cot)--dots 2-3-5 followed by dots 1-2-5-6,

This sign is used as follows:

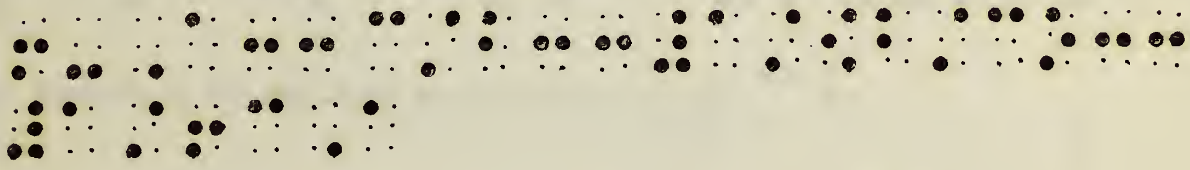
$$\cot A = \frac{b}{a} = \frac{1}{a/b} = \frac{1}{\tan A}$$



53. **SECANT** (sec)--dots 2-3-5 followed by dots 3-6,

This sign is used as follows:

$$\sec A = \frac{c}{b} = \frac{1}{b/c} = \frac{1}{\cos A}$$

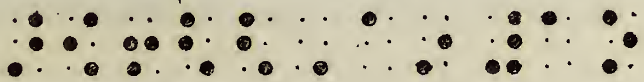


54. **COSECANT** (csc)--dots 2-3-5 followed by dots 1-2-6,

This sign is used as follows:

$$\cos A + \cos B = \sin (a+b) \csc C$$

$$\sqrt{\csc^2 A - 1}$$

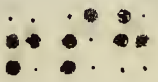


The index of the power to which a trigonometrical ratio is raised is, as is also the case in printed books, inserted between the sign of the ratio and the angle.

* In a sequence of related terms only the abbreviation for the first quantity is used.

Examples: $\sin^5 x = (\sin x)^5$

$\csc^2 x = (\csc x)^2$

55. **HYPERBOLIC SINE**--dots 2-3-5/2-3-4/1-2-5, 

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

56. **HYPERBOLIC COSINE**--dots 2-3-5/1-4/1-2-5,

57. **HYPERBOLIC TANGENT**--dots 2-3-5/2-3-4-5/1-2-5,

58. **HYPERBOLIC COTANGENT**--dots 2-3-5/1-2-5-6/1-2-5,

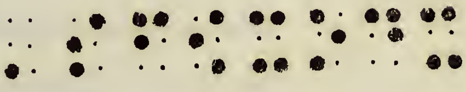
59. **HYPERBOLIC SECANT**--dots 2-3-5/3-6/1-2-5,

60. **HYPERBOLIC COSECANT**--dots 2-3-5/1-2-6/1-2-5,

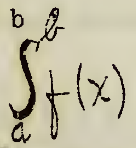
61. DIFFERENTIAL COEFFICIENT OF X WITH RESPECT TO T.

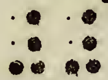
62. SECOND DIFFERENTIAL COEFFICIENT OF X WITH RESPECT TO T.

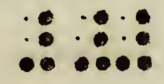
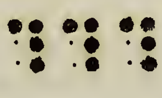
63. INTEGRAL SIGN--dot 3 followed by dots 2-3-4,

64. INDEFINITE INTEGRAL OF f(x) 

65. DEFINITE INTEGRAL OF f(x) BETWEEN THE LOWER LIMIT a AND THE UPPER LIMIT b



66. THE SIGN  IS USED TO DENOTE THAT ALL THE BRAILLE SIGNS OF LINE 1, WHICH FOLLOW IN THE SAME LINE OF THE PAGE, REPRESENT ARABIC NUMERALS.

67. THE SIGN  IS USED TO DENOTE THAT ALL THE BRAILLE SIGNS OF LINE 1, WHICH FOLLOW ON THE SAME PAGE, OR UNTIL THE SIGN  IS MET WITH, REPRESENT ARABIC NUMERALS.

68. DOLLARS ARE WRITTEN AS FOLLOWS:

$22

$45.50

SIGNS USED IN BRITISH AND AMERICAN MATHEMATICAL NOTATION--CONTINUED

ABBREVIATIONS APPEARING IN AMERICAN TEXTS

bushels----------bu

cents------------c

centimeter-------cm

cubic------------cu

degree-----------dg

dozen------------dz

feet-------------ft

gallons----------gal

hours------------hr

inches-----------in (not contracted)

miles------------mi

millimeter-------mm

minutes----------min* (in not contracted)

months-----------mo

ounces-----------oz

pound------------lb

quarters---------qr

dollars----------lower d

A period is placed after the above abbreviations when words stand alone.

angle------------dots 4-5-6 followed by letter a

circle-----------cl

denominator------denr

equation---------eqn

expression-------expn

fraction---------frn

graph------------dots 4-5-6 followed by letter g

negative---------neg

number--------------dots 4-5-6 followed by letter n

numerator-----------numr

perpendicular-------perp

point---------------dots 4-5-6 followed by letter p

positive------------pos

question------------qn

quantities----------qnts

value---------------dots 4-5-6 followed by letter v

* Sometimes written mn.

SIGNS USED IN BRITISH AND AMERICAN MATHEMATICAL NOTATION--Continued

ABBREVIATIONS USED IN BRITISH TEXTS

The following abbreviations are used when immediately preceding the numeral sign. In cases of sequence the first abbreviation only is used:

LONG MEASURE

inches----------in* numeral sign

feet------------ft " "

yards-----------yd " "

poles-----------pl " "

furlongs--------fur " "

miles-----------m " "

leagues---------lg " "

SQUARE MEASURE

square inches------in*, dots 1-2-6, numeral sign

square feet--------ft, dots, 1-2-6, numeral sign

square yards-------yd, dots 1-2-6, numeral sign

square poles-------pl, dots 1-2-6, numeral sign

roods--------------rd,numeral sign

acres--------------ac,numeral sign

square miles-------m, dots 1-2-6, numeral sign

CUBIC MEASURE

cubic inches-------in*, dots 1-4-6, numeral sign

cubic feet---------ft, dots 1-4-6, numeral sign

cubic yards--------yd, dots 1-4-6, numeral sign

* Contraction not used.

TROY WEIGHT

grains--------gr numeral sign

pennyweights--dwt " "

ounces--------oz " "

pounds--------lb " "

AVOIRDUPOIS WEIGHT

drams---------dr numeral sign

ounces--------oz " "

pounds--------lb " "

stones--------st* " "

quarters------qr " "

hundredweights-cwt " "

tons----------tn " "

APOTHECARIES WEIGHT

grains--------gr numeral sign

scruples------scr " "

drachms-------dr " "

ounces--------oz " "

pounds--------lb " "

DRY MEASURE

gills---------gi numeral sign

pints---------pt " "

quarts--------qt " "

SIGNS USED IN BRITISH AND AMERICAN MATHEMATICAL NOTATION--Continued

ABBREVIATIONS USED IN BRITISH TEXTS

DRY MEASURE--Continued

gallons----------gl numeral sign

pecks------------pk " "

bushels----------bsh " "

quarters---------qr " "

loads------------ld " "

CLOTH MEASURE

inches-----------in* numeral sign

nails------------nl " "

quarters---------qr " "

Flemish ell------letter f, dot 3, letter e, numeral sign

yards------------yd numeral sign

English ell------letter e, dot 3, letter e, numeral sign

French ell-------fr, dot 3, letter e, numeral sign

PAPER

sheet------------sht numeral sign

quires-----------qe " "

reams------------rm "" "

ALE

pints------------pt numeral sign

quarts-----------qt " "

gallons----------gl " "

firkins----------fn " "

kilderkins-------kn " "

barrels----------bl " "

* Contraction not used.

hogsheads-------hhd numeral sign

puncheons-------pn " "

butts-----------bt " "

WINES

pipes-----------pp numeral sign

MONEY

pounds----------l numeral sign

shillings-------s " "

pence-----------d " "

francs----------fr " "

centimes--------ct " "

dollars---------lower d numeral sign

TIME

seconds---------sc numeral sign

minutes---------mn numeral sign

hours-----------hr " "

days------------dy " "

weeks-----------wk " "

months----------mnth " "

years-----------yr " "

SIGNS USED IN BRITISH AND AMERICAN MATHEMATICAL NOTATION--CONTINUED

ABBREVIATIONS USED IN BRITISH TEXTS

MISCELLANY

degrees-----------deg numeral sign

dozen-------------doz " "

horse-power-------h, dot 3, p, numeral sign

links-------------lk numeral sign

chains------------chn " "

fathoms-----------fth " "

nautical miles----n, dot 3, m, numeral sign

knot--------------kt numeral sign

METRIC SYSTEM

millimetres-------mm numeral sign

centimetres-------cm " "

decimetres--------dm " "

metres------------mt " "

decametres--------dots 4-6,* dm, numeral sign

hectometres-------hm numeral sign

kilometres--------km " "

milligrammes------mg " "

centigrammes------cg " "

decigrammes-------dg " "

grammes-----------gm " "

decagrammes------dots 4-6,* dg, numeral sign

hectogrammes-----hg numeral sign

kilogrammes------dots 4-6,* g, numeral sign

millilitres------ml numeral sign

centilitres------cl " "

decilitres-------dl " "

litres-----------lt " "

decalitres-------dots 4-6,* dl, numeral sign

hectolitres------hl numeral sign

kilolitres-------kl " "

* Dots 4-6, the former British capital sign, is used to distinguish the decametres from the decimetres. This distinction is carried through the system.

IMPORTANT POINTS TO REMEMBER IN THE WRITING OF BRAILLE MATHEMATICAL NOTATION

1. When an algebraical quantity represented by a Braille sign of Line 1 (see page 2) has a numerical coefficient the sign, dots 1-6, must be inserted after the number; in other cases it may be omitted.

2. When the numerator or the denominator of a fraction consists of two or more terms it must, in Braille, be enclosed in brackets; should bracketed portions already exist, a second pair must be introduced. (A term is a or b in a + b; ab or cd in ab - cd.)

3. When the whole of an expression (algebraical or numerical) consisting of two or more factors or terms is to be affected by either an index or root sign, it must be enclosed in brackets; if any part of such expression is already bracketed, a second pair should be introduced. (A term is defined in the preceding paragraph. A factor is the 6 or w in 6w.)

Note. In writing $\frac{ab}{cd}$ brackets are not necessary because, in the writing of fractions, brackets are used only when the numerator or the denominator consists of two or more terms. In writing the square root of ab, brackets are necessary because of the fact the part affected by the root sign consists of two or more terms or factors.

IMPORTANT POINTS TO REMEMBER IN THE WRITING OF BRAILLE MATHEMATICAL NOTATION--Continued

4. The index of the power to which a trigonometrical ratio is raised is, as in printed books, inserted between the sign of ratio and the angle.

5. The numeral sign is not required after the numerical index, numerical suffix, nor the factorial sign.

6. The root sign includes no number but the signs for the square and cube roots include the numbers 2 and 3 respectively. (This does not apply to the writing of the cube root on the Calculating Board.)

ADDITIONAL SIGNS REQUIRED IN THE WRITING OF BRAILLE MATHEMATICAL NOTATION

The dotted signs are those which do not appear in print but must be inserted in Braille.

$2a + 4b$ in Braille must be written $2 \times a + 4 \times b$

$2x + 4y$ does not require the multiplication sign as only the signs of Line 1 are affected by the numeral sign and used for numbers.

$$\sqrt{23} \qquad \sqrt{\left(\frac{2}{3}\right)} \qquad \sqrt{a^3} \qquad \sqrt{(a - b)}$$

$$\frac{(a - b)}{(x - y)} \qquad \frac{(2 \times a - 3 \times b)}{(2x - 3y)} \qquad \frac{ab}{(x - y)}$$

$$\frac{\left(bd\left(\frac{a}{b} - 3\right)\right)}{\left(bd\left(\frac{c}{d} - 5\right)\right)} \qquad \sqrt{(2(a + b))} \qquad \sqrt{(2 \times a)}$$

$$\sqrt{\left(\frac{(1 - \cos A)}{2}\right)} \qquad \pi h^2 \left(\frac{(c^2 + 4h^2)}{8h} - \frac{h}{3}\right)$$

$$\frac{(1 - \cos A)}{\left(2 + \sqrt{(2(1 - \cos A))}\right)} \qquad x^{(m+1)}$$

COMPACT LIST OF BRAILLE MATHEMATICAL SIGNS ALPHABETICALLY ARRANGED

WRITER		SLATE
	Brackets	
	"	
	"	
	"	
	"	
	Cosecant	
	Cosine	
	Cotangent	
	Cube	
	Cube root	
	Dashed letters (followed by)	
	Decimal sign	
	Difference between	
or	Divided by	or
	Dollar sign	
	Factorial sign	
	Fourth power	
	Greek letters (preceded by)	
	Hyperbolic cosecant	
	Hyperbolic cosine	
	Hyperbolic cotangent	
	Hyperbolic secant	
	Hyperbolic sine	
	Hyperbolic tangent	
	Infinity	
	Integral sign ∫	
	Is equal to =	
	Is equal to or less than ≦ or ≮	
	Is greater or less than	
	Is greater than >	
	Is greater than or equal to ≧ or ≯	
	Is greater than, equal to, or less than	
	Is identically equal to ≡	
	Is less than <	
	Is nearly equal to	
	Is not equal to ≠	
	Is not greater than	
	Is not less than	

COMPACT LIST OF BRAILLE MATHEMATICAL SIGNS ALPHABETICALLY ARRANGED

WRITE		READ
	Brackets	
	"	
	"	
	"	
	Constant	
	Cosine	
	Cotangent	
	Cube	
	Cube root	
	Dotted letters (followed by)	
	Decimal sign	
	Difference between	
	Divided by	
	Dollar sign	
	Factorial sign	
	Fourth power	
	Greek letters (preceded by)	
	Hyperbolic cosecant	
	Hyperbolic cosine	

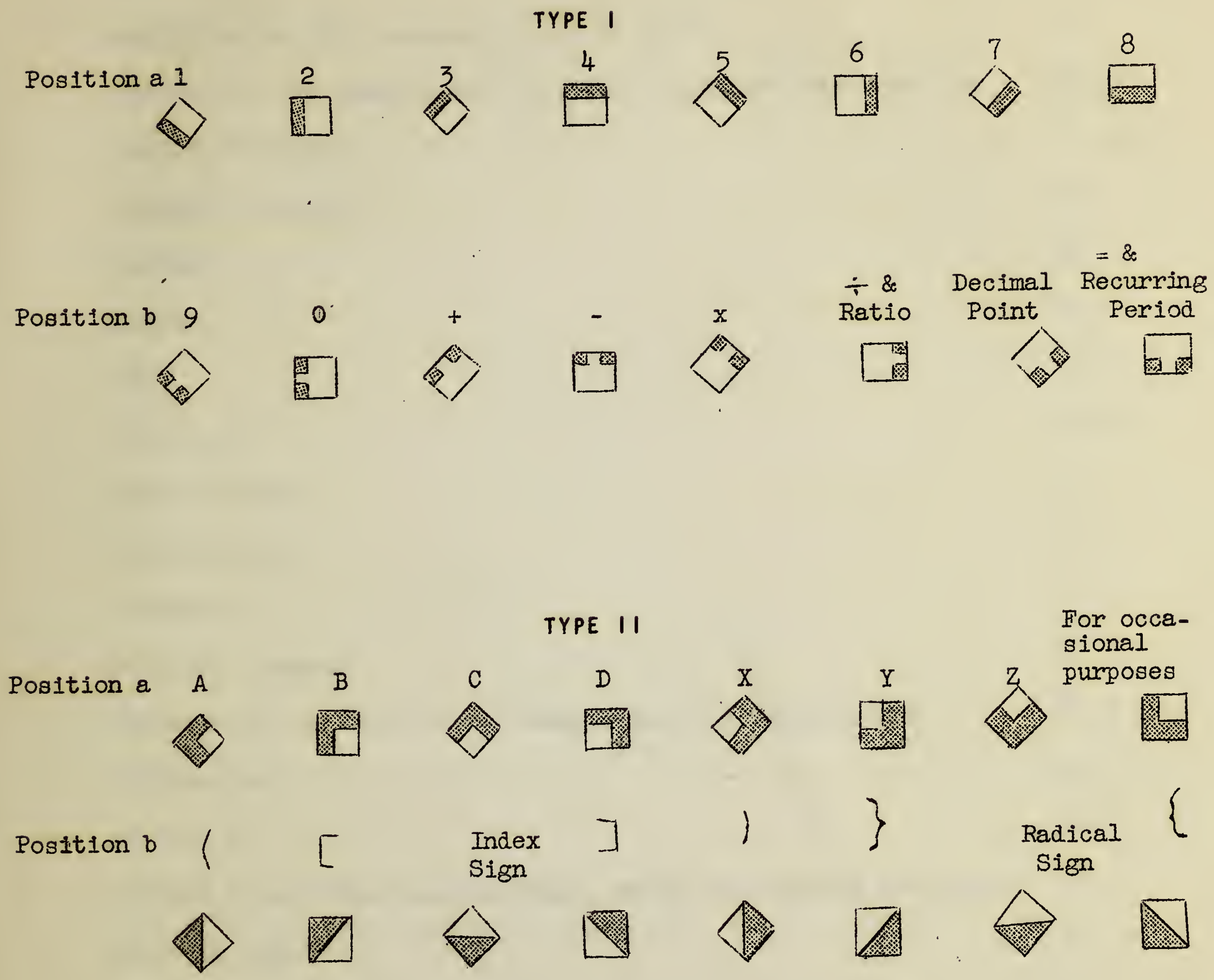
WRITE		READ
	Hyperbolic cotangent	
	Hyperbolic secant	
	Hyperbolic sine	
	Hyperbolic tangent	
	Infinity	
	Integral sign	
	Is equal to	
	Is equal to or less than ≦ or ≤	
	Is greater or less than	
	Is greater than	
	Is greater than or equal to ≧ or ≥	
	Is greater than, equal to, or less than	
	Is identically equal to	
	Is less than	
	Is nearly equal to	
	Is not equal to	
	Is not greater than	
	Is not less than	

COMPACT LIST OF BRAILLE MATHEMATICAL SIGNS ALPHABETICALLY ARRANGED--Continued

WRITER SLATE

- letter sign
- Literal index
- Literal suffix
- Logarithm to base 10
- Long division (after divisor)
- (before quotient)
- Minus −
- Minus or plus ∓
- Multiplied by
- Numerical index
- Numerical suffix
- Plus +
- Plus or minus ±
- Proportion
- Ratio
- Recurring period
- Root sign
- Square
- Square root
- Varies as ∝

THE TAYLOR ARITHMETICAL AND ALGEBRAICAL CALCULATIONS



For Trigonometry, the first six positions of Type II (b) may be used respectively for Sine, Cosine, Tan., Cosec., Sec., Cotan.

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